Kondratiev Cycle Modelling

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***Abstract*—Economic activity cycles differ in their nature from several rather short 3,7-11, and 25-year cycles arising from the investment process to long cycles of another nature. According to a wide-spread explanation Kondratiev cycles are somehow driven by innovations. We introduce a formal model where at a lower level each moment is observed bi-stable general equilibrium sub-system, potentially having equilibria of zero wages (labor-excessive case) and substantially non-zero wages (labor-deficit case), while at the upper level, we introduce wage-dependent technological progress influencing the labor consumption that leads to periodical disappearing of current stable equilibrium due to competence of neutral and labor-saving technological progress pushing labor productivity up and down in labor-deficit and labor-excessive cases respectively. Due to that, we observe a robust Kondratiev cycle.**

Keywords—Kondratiev cycle, technological progress, labor excess, rigid cycle.

I. Introduction

Kondratiev cycle was first recognized in the Conjuncture Research Institute of N. Kondratiev [1,2,6,7]. Yet from that time there formed no common view of why do these cycles arise.

Some researchers believe that there is a wide-range time hierarchy of economic activity cycles from at least 1 year to several centuries or even a thousand or thousands of years for example Forrester (200 years) and Toffler (more than 1000 years) cycles.

The more or less common view is that we observe more than a year time length cycles of 3, 7, 11, 25 years, and more longitude. We do not write here the length of Kondratiev's cycle since it's believed to be not constant and shortened at a time still consisting of an integer number of 11-year cycles. The length observed at the beginning of the XX century was estimated as at least 40-50(60) years[5,7]. All 3,7, 11, and 25-year cycles should be attributed to investment cycles the different stock types (with different lifetimes) are responsible for. The current or circulating assets produce a 3-year cycle named after Kitchin, and the equipment depreciation produces a 7-11-year cycle named after Juglar. The Kuznets building-construction cycle has approximately a 25-year length according to normal building lifetime (though the percentage rate addition to the depreciation intensity here already dominates). As for Kondratiev's cycles, there is no common understanding. We admit one observe long-leaving objects in the economy (like power plants and roads as well as some part of buildings - of course, if we omit percentage rate addition), and that could partially explain Kondratiev's cycle as well. Still, we consider our own explanation as more adequate: endogenous technological progress (leading to labor-saving) driven by labor-force price, while the last depends on the current excessive demand for labor-force (depending on labor-saving success or unsuccess).

In the short words, we believe that technological progress is not neutral. We admit, that there is a steady flow of critical innovations, that cannot itself lead to any cyclicity. But we believe, that labor-saving is not steady and acts faster in times of labor deficit, when labor is expensive, why slow down at times of hard labor excess when wages are close to nearly zero at most territories (if a special aspect is taken into account too). Further, we consider a potentially bi-stable economic system with potentially two zero- and non-zero wage equilibria depending on the labor-saving rate. Labor-saving affects labor demand via final consumption that is either wage-dependent.

II. The Structure Of The Model

Using the consequent bifurcation diagram as a slow manifold (main isocline of the fast wage-price and funds equilibrium sub-system) we add an equation of labor-saving progress, depending on the wage level. Thus, we get an upper-level two-dimensional system, with at least a 3-dimensional subsystem of wages, prices, and output (or industrial output in our case), found at constant technologically defined labor consumption per product unit.

III. Labour Excess Phenomenon In Different Economies

In works [3,4] we have discussed how labor excess arises in industrial, agro-industrial, and agricultural economies. Here we use an agroeconomic model with a relatively weak labor excess. This model acts as a basis qualitatively not far from the reality at the beginning of the XIX century, when all the countries were primarily agrarian, except for some parts like the "capital London" region (known as contemporary England) in the British Empire. In our model the labor excess is used in the industry. In the case of any non-zero unemployment, the wages are (for brevity) zero. If the industry demand exceeds the agricultural labor excess the wages grow until one of two cases happens

1) the industry labor-demand with the wage growth has become equal to the agricultural labor excess

2) the wage has achieved the level of limited productivity of agriculture (how this is calculated we discuss in [4]) and then the unbounded amount of labor may be given at this "fixed" price.

As it earlier announced, we will get a model where several equilibria are possible.

Exactly the same qualitative result is possible for the contemporary industrial society [3]

IV. Agricultural Labour Excess Phenomenon

The purely agricultural model is fundamentally based on Fernand Braudel's verbal descriptions and explanations is deeply discussed in [-] and mainly in [4]. We will reproduce here some most necessary facts and conclusions.

1) The agricultural society economy on a homogeneous landscape with two main technologies may be well enough described by a linear programming problem with a system of three linear semi-rigid constraints of inequality type with three more trivial ones, requiring positivity (non-negativity) for the integral population and territory shares these technologies use.

In the equilibrium of landscape and population we maximize population in another case (the new countries like the USA, Australia, Canada, etc.) the population is an additional equality-type constraint (while something like net product (but not exactly) may and should be maximized).

Thus, according to [4], we get three equilibrium types and one purely labor-deficit un-equilibrium type of agrarian society differing in wage and main agrarian technology (or their optimal mix).

The most interesting is an equilibrium type, which may be described in the following terms.

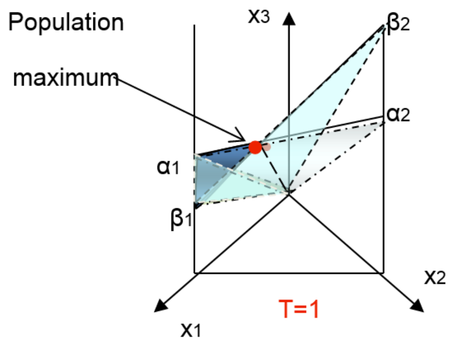

Fig. 1. The typical system of resource constraints for an agrarian economy (the equilibrium labor-deficit type). This (further type II) corresponds to the northern Europe (a wealthy North)

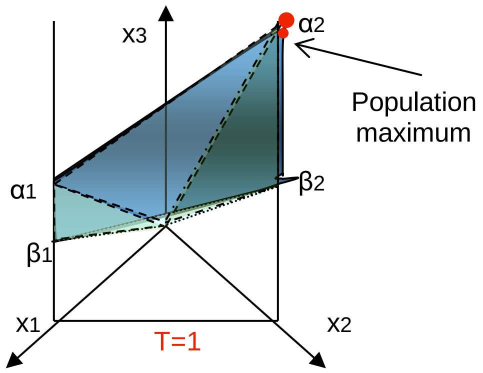

Fig. 2. The typical system of resource constraints for an agrarian economy (the equilibrium labor-exessive type for superproductive crop production in southern countries). This (further type I) corresponds to the global South (a Poor South as it is often called) including countries from Egipt, India, and China, to Southern Europe including Spain, Italy, and, even, France.

In that economy only two technologies are available: (index 1) cattle breading and (further index 2) crop production. Each technology gives food in an amount that permits to feed αx families and at the same time requires labor resources corresponding to the number of βx families.

If we dial with homeostasis the territory S is optimally used at some technology

$x_1 + x_2 = S$ or for the general case

$x_1 + x_2 \le S$. (2)

If the population consists of N families we can describe the resource-planning problem in the form

$$\begin{cases} x_1 + x_2 = S \\ \beta_1 x_1 + \beta_2 x_2 \le N \\ \alpha_1 x_1 + \alpha_2 x_2 \ge N \\ N \ge 0, x_1 \ge 0, x_2 \ge 0 \end{cases}, N \to \max \qquad (3)$$

So far there is a parametrical area, where labor is deficit and an area where it is excessive.

Further, we assume that industry consumes a small excess of labor existing in the weakly labor-excessive economy of type I (Fig.2) in the best case making it analog to type II (Fig.1) due to the upward shift of the labor-consumption constraint

$$\begin{cases} x_1 + x_2 = S \\ \beta_1 x_1 + \beta_2 x_2 + IndustryLaborConsumption \le N \\ \alpha_1 x_1 + \alpha_2 x_2 \ge N \\ N \ge 0, x_1 \ge 0, x_2 \ge 0 \end{cases},$$

$N \to \max$

The industry labor consumption above further is formally described by the product

$Q\beta_{\Pi}$,

where Q - the industrial output, and β_{Π} is a labor consumption per output unit.

That means industry labor consumption should be more than

$L_0 = \alpha_2 x_2 - \beta_2 x_2$,

that is the excessive labor agrarian sector

$IndustryLaborConsumption = \beta Q \ge \alpha_2 x_2 - \beta_2 x_2$

V. The Spacial Aspect

Let us introduce and denote

11 – labor-excessive crop-productive

12 – deep labor-excessive cattle-breading

13 – weakly labor-excessive hunter-gatherers agriculture

2 – mixed (crop productive plus cattle-breading)

Nonequilibrium (generally so-called immigrant)

3(1)– nonequilibrium (over labor-excessive)

(practically the labor deficit substantially depends on the property access regime)

4 – unproductive (acritical and other life-free deserts)

5 –variable complex landscape (altitudinal zonality)

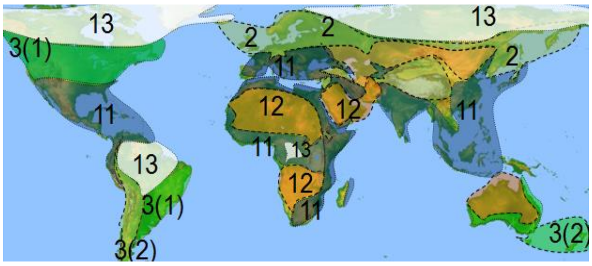

Fig. 3. Social-economic Agricultural types in the XIX century.

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VI. The Number of Products In An Industrial Extention of The Agrarian Model

Here we assume that there are several consumption alternatives despite one product model. In the simplest case, we may consider three ways of income spending. Current consumption (food and elite products) and saving or potential future consumption (in terms of land aggregation mainly). To simplify we may talk in terms of trade between food and future consumption.

Exactly this approach we will use in the agro-industrial case.

VII. The Structure of The Integral Model

The agrarian model above is a screen of the consequent agro-industrial model exhibiting conjuncture bi-stability driven via a labor market. Over that model we will build a semi-neutral technical progress model, reflecting permanent improvement of new goods productivity depending on the wage scale.

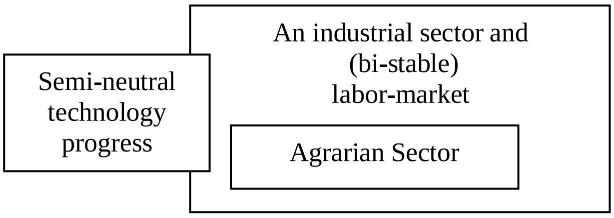

Fig. 4. The 3-layer integral model structure: starting from a purely agrarian linear two-technology model we come to an agroindustrial one and after that apply dependence of labor industrial productivity from the wage scale obtaining a rigid-cycle dynamics of conjuncture waves.

VIII. The Consumption Preferences (The Utility Function)

Let us assume, that there are three products: agricultural A, elite(property) R, and industrial Q. The utility is given by a complex function, that can be approximated as

$$U = R^{\alpha_R} \cdot Q^{\alpha_Q} \cdot A^{\alpha_A}$$

with radically different coefficients for the labor force (with no income but their wage in a competitive labor market) and property-holders with rent-like income from property.

So, we have two vectors $(\alpha_R, \alpha_Q, \alpha_A)$: $\alpha_R + \alpha_Q + \alpha_A = 1$.

$(\alpha_R^w, \alpha_Q^w, \alpha_A^w)$ for workers and $(\alpha_R^e, \alpha_Q^e, \alpha_A^e)$ the elite.

$\alpha_R^w \approx 0,\ \alpha_Q^w >> \alpha_Q^e,\ \ \alpha_A^w >> \alpha_A^e$.

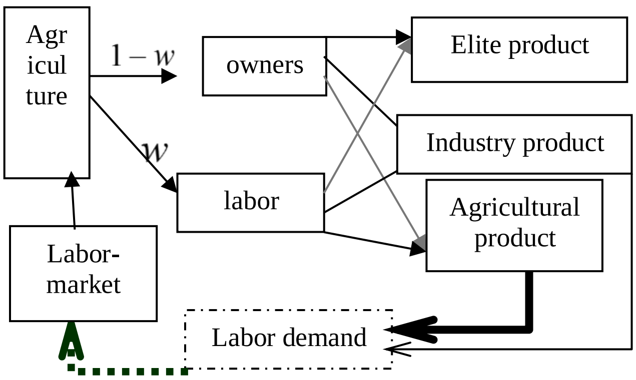

Fig. 5. Approximate financial flow scheme.

We assume that property-holders have much more income than workers and the share of both industrial and agricultural products in their consumption is much less than the same one for workers.

IX. Technological Set

The product factor consumption is given by the matrix

If we assume R as land (a factor of nature)

$$\begin{array}{lcccc}
 & \multicolumn{3}{c}{\textit{factor consumption}} & \textit{productivity} \\
R_{property\ \&\ elite} & 0 & 0 & p_{LAND} & 1 \\
Q_{industry} & 0 & \beta_\Pi & p_0 & 1 \\
A_{CattleB.} & 1 & \beta_1 & 0 & \alpha_1 \\
A_{crop.p} & 1 & \beta_2 & 0 & \alpha_2 \\
factor & land\ S & labor\ L & food\ A &
\end{array}$$

It is crucially important, that elite-product consumption growth does not directly lead to labor-force consumption growth as the property-holders spend their excessive income to buy the land from the poor workers. We may think of them buying it for food (at least as a universal price measure). In the last case, one may put at the 3rd position of the first string a land price (may be expressed in the year crops can feed a family if we talk of a corresponding square).

If we assume R as some import product the first coefficient line is slightly different:

$$\begin{array}{lcccc}
 & \multicolumn{3}{c}{\textit{factor consumption}} & \textit{productivity} \\
R_{property\ \&\ elite} & 0 & 0 & 1 & 1 \\
Q_{industry} & 0 & \beta_\Pi & p_0 & 1 \\
A_{CattleB.} & 1 & \beta_1 & 0 & \alpha_1 \\
A_{crop.p} & 1 & \beta_2 & 0 & \alpha_2 \\
factor & land\ S & labor\ L & food\ A &
\end{array}$$

If the land is scarce then the situation here is the same: nobody can produce more agricultural product.

Initially, the Industrial Revolution took place in the strict labor deficit type (the English core of the British Empire and Farther in Northern Europe, which had a semi-labor-excessive type [1]), but one should take into account, that the whole market always included many labor-excessive countries either, so the whole economy was at least partially labor-excessive, that’s why we build our agro-industrial model at the basis of weekly labor-excessive type I took as an economic background.

The reason one should start from an agricultural background is that until the XIX century, there were no countries with the domination of industry although we can point to at least one province (England itself) that became a small industrial center of a huge British Empire and even in the contemporary world approximately half of the economy remains agrarian. So far until the XX century, the industrial economy remained a small correction to dominated agricultural surrounding background.

That’s why, further we follow the scheme in Fig.2: the income in the agricultural sector is subdivided into two parts. Part 1 equals w - the workers' share (worker wage) and part 2 is its addition to obtain 1: $1-w$ the owner's share. The worker's share may become approximately zero, while the small class of owners this case gets all the income.

The preferences of workers and owners should be nearly the same, but due to their income difference, it's easier to consider them different in the linear approximation used below.

For simplicity one may consider worker's consumption linearly divided between industry product and agricultural product (each using much labor per product unit), while high-income owners consume much more elite product with nearly zero shares of labor (in an open economy we may even consider elite product as an import with zero labor share, while in the primitive closed economy, we may consider rental real estate assets as a considerable way of excessive elite saving and spending with either approximately zero labor share in natural - land-like assets). For the technological matrices above we get the fallowing system

$$\begin{cases} p = p_0 + \beta_\Pi w \\ w = Max(\min({}^1\!/_\varepsilon \, (Q_d \beta_\Pi - L_o, \dfrac{\Delta\alpha}{\Delta\beta}),0) \\ Q_d = \dfrac{\left(\alpha^w{}_Q \cdot w + \alpha^e{}_Q \cdot (1-w)\right) \cdot L}{p_Q} \end{cases} \quad \begin{cases} \tau_{\beta_\Pi} \dfrac{d}{dt} \beta_\Pi = -\chi(w/w_0) + (\beta_0 - \beta_\Pi) \\ \begin{cases} \tau_p \dfrac{d}{dt} p_Q = (p_0 + \beta_\Pi w) - p_Q \\ \tau_w \dfrac{d}{dt} w = Max(0; \min({}^1\!/_\varepsilon \, (Q_d \beta_\Pi - L_0); \dfrac{\Delta\alpha}{\Delta\beta})) - w \\ \tau_Q \dfrac{d}{dt} Q_d = \dfrac{\left(Z^w{}_Q \cdot w + Z^e{}_Q \cdot (1-w)\right) \cdot L}{p_Q} - Q_d \end{cases} \end{cases}$$

That's why we obtain from one to three equilibria only two (or one) of them can be stable. In the upper (non-zero) equilibrium the labor price should be equal to the corresponding case of a purely agrarian economy with a weak labor deficit done by $w_a = \dfrac{\Delta\alpha}{\Delta\beta}$, while in the case of initially zero-wage [4], $w = 0$

the corresponding labor demand needed to admit only owners' preferences may be much less than the available labor supply, which fixes the zero labor price. In this case we at the same time have two stable equilibria with the intermediate unstable is characterized by wage at the level $w_{UnSt} = \dfrac{L_o p_0 - \alpha^e{}_Q \cdot L}{L \cdot (\alpha^w{}_Q - \alpha^e{}_Q) - L_0 \beta_\Pi}$ that is either a level of a single stable equilibrium in a single equilibrium case when

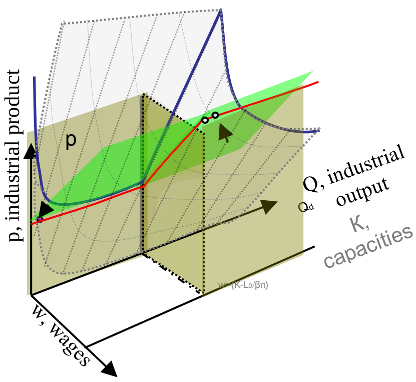

Fig. 6. The industrial supply (sloped flat), and labor supply (3D step-function) depend on output (connected with the labor demand), and industrial product demand depends on wage. The situation when low zero-wages equilibrium at high β_Π disappears (merging the unstable one), that means the low phase of the Kondratiev cycle termination and switching to a long growth phase.

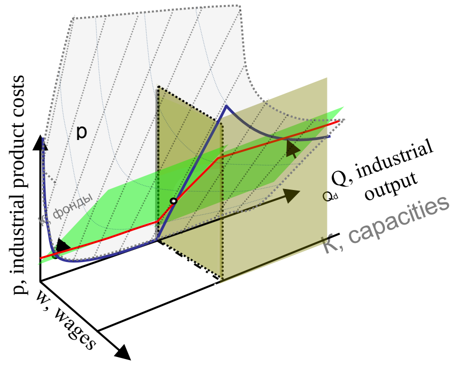

Fig. 7. The typical situation when low zero-wages equilibrium coexists with high non-zero wages equilibrium, that means the system may be in both phases – depending on the pre-history.

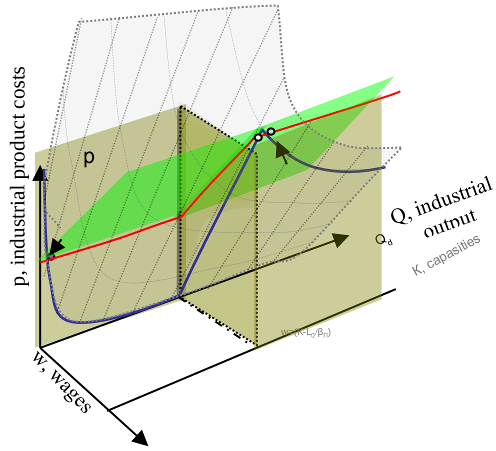

Fig. 8. The situation when high non-zero-wages equilibrium at low β_Π disappears (merging the unstable one), that corresponds the high phase of the Kondratiev cycle termination and switching to a crisis (or depressional) phase.

X. Semi-Neutral Technical Progress Model

The key labor-consumption coefficient β_Π - the main parameter, controlling bifurcations in the model above in the long run model of the Kondratiev cycle becomes a variable. It depends on two natural processes

1) the neutral technological progress assuming crucial innovations. It's pretty well known, that such innovation flow is time-stable and does not depend on economic factors

2) labor-consumption or labor-saving: as soon as a new (or not new) product or way of production appears there may be more or less stimulus for labor-saving. If we see large labor resources able to work for food only, then we have very small incentives for labor-saving. This circumstance has a wide spectrum of evidence: in nearest times – at late 1980 the main part of industrial robots was situated in three countries: Eastern Germany, USSR, and Japan, which were closed and faced deep labor deficit.

In an agrarian world (see our last section on the spacial aspect) capitalistic institutions, as well as critical innovations, were widespread but we hardly ever saw a collection of labor-saving innovations, especially with implementation in capital-intensive or some resource-consuming technologies.

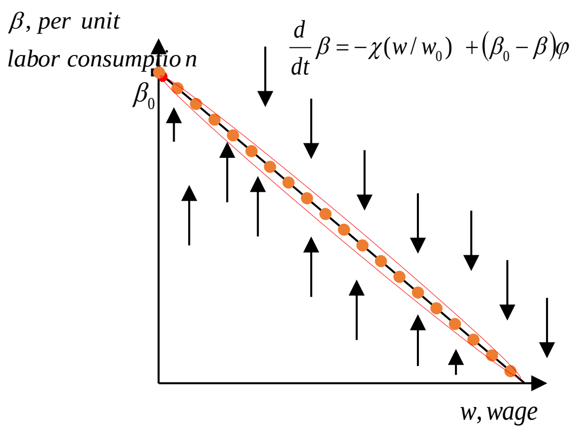

Fig. 9. The dynamical equation of tecnological progress. The labor-consumption coefficient β_{Π} becomes smaller and smaller as soon as wages rise up and vice-versa.

Let's introduce the following phenomenological equation

$$\frac{d}{dt}\beta = -\chi(w/w_0) + (\beta_0 - \beta)\varphi$$

reflecting lower labor consumption equilibria at higher wages: Fig.9 ($\chi, w_0, \varphi, \beta_0$ - coefficients, β, w - variables).

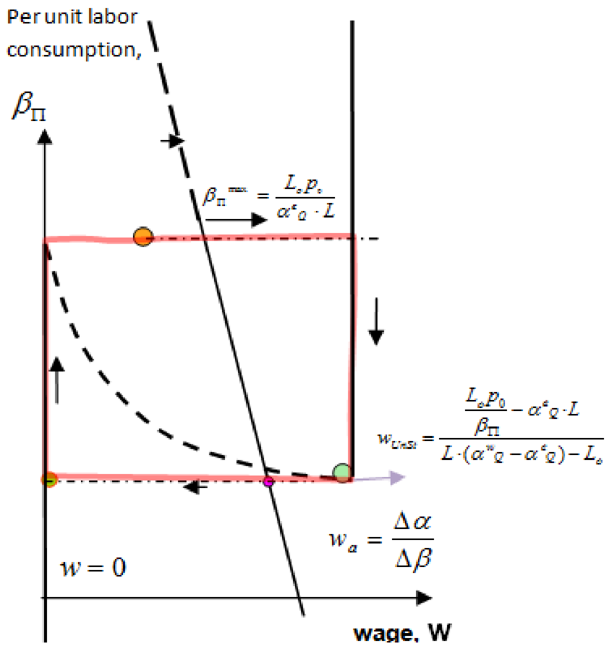

Fig. 10. The upper-level phase diagram othe f the Kondratiev cycle. Horizontal line – real wages (in food units) the upper line is labor consumption (the value inverse to labor-productivity). The vertical semi-straight lines are the stable parts of the low-level agro-industrial general equilibrium corresponding to the zero and non-zero wages. The last one also corresponds to the agrarian mixed-type productivity. The dashed line is a branch of unstable intermediate equilibria. They form a system of fast isoclines the upper-level system slowly walks by until the stable branch terminates. The sloped straight-line is the slowest technological progress isocline exhibiting the direction the system slowly moves by the fast system isoclines (vertical semi-straight lines).

Then general system can have the phase diagram described in Fig.10 with cyclical movement along the two semi-straight-line manifolds with fast switching when they lose stability at their terminal points as shown in Fig.6 (turn from depression to growth) and Fig.8 (return to depression after β_{Π} decline).

Talking about weakly labor-excessive point model we to apply it to reality should assume that there is an interaction between at least two regions in Fig.3 the semi-labor deficit Northern Europe and ultra-labor deficit Anglo-Saxon world at one side and its neighborhood like the global South including South and South-East Asia, South Europe and less important Africa and it's a problem of our own preference whether we talk about movement of regional boarder of labor-excessive and labor-deficit regions or on the contrary we for simplicity consider a mixture of labor deficit core region and it nearest periphery that can correspond to weakly labor excessive region we have described in our point model.

We may also assume another center-periphery effect due to the transportation costs rising from the center (sea coast) to the periphery (Hartland) the situation described at Fig. 6-8 when transportation cost acts as an addition to p_0 rising from center to periphery.

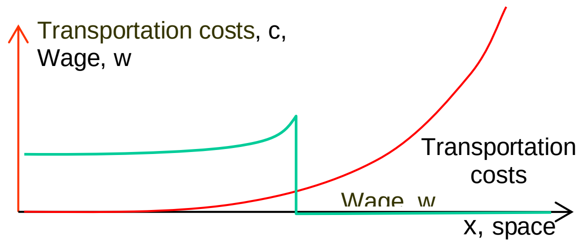

Fig. 11. Spacial system of the type we regard below. The space effects via transportation costs are included in the non-wage expenses p0 we introduce later.

At some moment (distance to the center) we see the threshold (or bifunctional) situation like in Fig.8, that means, that further we see only depressive (labor excessive) periphery. Due to the Fig.10 limit cycle this either assumes labor-deficit island size (radius of bifurcation as at Fig.8) pulsation with cyclical β_{Π} (per unit industrial labor consumption) dynamics.

Of course, at current situation the share of agricultural sector in full labor consumption is negligible in industrial countries, so far, a new approach to Kondratiev cycle description is required. Still, until the middle of XX century it wasn't so nowhere and even now about a half of world population lives in nearly agricultural economies.

XI. Conclusion

We have demonstrated a formal Kondratiev cycle model. We used a hierarchy of regional-dependent models:

1) agrarian one, that points the labor deficit core where labor-saving innovations tend to store and develop,

2) on this basis we could talk of weakly-labor deficit core including its nearest periphery exhibiting bi-stability after including an industrial branch,

3) then we have introduced a labor consumption dynamic finally led to a long cyclical behavior.

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