Dynamic programming approaches for single-track scheduling problem

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We consider a scheduling problem with the single railway track connecting two stations. Single railway tracks constitutes the major part of railway transport networks in many regions, and are very common in supply chains. Survey of the railway planning models and methods can be found in the publication of Lusby et al [1]. Usually problems are considered in terms of scheduling theory as job-shop problems, and dynamic programming approach [2] or heuristic method [3] is applied.

We investigate the problem where trains travel between two stations, denote them as Station 1 and Station 2, which are connected by a single track. Each train travels either from Station 1 to Station 2 or from Station 2 to Station 1. The transportation commences at time $t = 0$. Define the set of all trains at both stations as N. Two models are considered: the model with the siding on a track and the model without a siding. For both models we propose the solution algorithms based on the dynamic programming method.

The model without a siding is described as follows. There are α different possible speeds of trains, where α is a rather small number. For example, we can divide trains into 3 groups according to the speeds: freight trains, passenger trains, express trains. Each train travels with one of constant α speeds, for each speed the train traversing time for the track is given. There must be a minimal safe distance between two trains simultaneously moving in one direction, so a minimal time interval between the departure of two trains is required, it depends on the speeds of successive trains. In addition, train movement on the track is possible only in restricted set of time intervals $V = \{t | t \in [u_1, v_1] \cup [u_2, v_2] \cup$

$\ldots \cup [u_q, v_q]\}$, where q is a number of intervals. We consider the set of objective functions which can be represented with as special general form. For each train some additional parameters depending on objective function are given. In the most general case of the problem with α speeds of trains, given possible interval of movement V and λ sets of trains with specified departure order the solution algorithm constructs optimal schedule in $O(q^2 \log q n^{2\alpha^2+2\alpha+1} n^{\lambda} \log n)$ operations, where $n = |N|$ is the number of trains on both stations at the initial moment. In some cases it is possible to significantly reduce the complexity, if $\alpha = 1$, $V = \{t | t \in [0, \infty)\}$, and objective function is maximum lateness, then the complexity is $O(n^2)$.

In the second model there is the siding on the track, which capacity is one train. On the track it is possible to cross two incoming trains only in the siding. All trains have equal constant speed, train traversing times for track segments separated by the siding are given. Minimal time interval is required between the departures of two trains from one station and between the arrival of two trains to the siding. For each train $i \in N$ due date d_i or priority coefficient w_i are given. Two objective functions which have to be minimized are considered: maximum lateness and the weighted sum of arrival moments. For each objective function the complexity of solution algorithm is $O(n^2)$.

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