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Short communication

## A note on the paper 'Single machine scheduling problems with financial resource constraints: Some complexity results and properties' by E.R. Gafarov et al.

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## ABSTRACT

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In the article E.R. Gafarov, A.A. Lazarev, F. Werner, Single machine scheduling problems with financial resource constraints: Some complexity results and properties, Mathematical Social Sciences, 62 (2011), 7–13, the following mistake is found in Section 3.2, where the authors consider the problem denoted as 1|NR,  $d_j = d$ ,  $g_j = g | \sum T_j$  and claim that it is NP-hard. In the proof, a reduction from the Partition Problem was used which is not polynomial, since *M* exponentially depends on *n*.

However, it is not difficult to correct this proof. The main idea of using  $M^{n-i+1}$  was that the processing time of a job

belongs to a pair with the smallest number being greater than the total sum of the processing times of all jobs from the pairs with larger numbers, e.g., for the job  $V_2$ :  $p_2 \gg \sum_{i=2}^{n} (p_{2i-1} + p_{2i})$ .

Instead of using  $p_{2i} = M^{n-i+1}$ , where  $M = (n \sum b_j)^n$  (see the definition of the instance given in (3) on page 11), we can consider, e.g.,  $p_{2i} = 2n \cdot 2^{n-i+1}M$ , where  $M = (n \sum b_j)$ . In this case, the reduction will be polynomial in the input length, if we suppose that all digits used are coded in a binary system with approximately  $2^n$  zero–one symbols per digit.





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