Short communication

# A note on the paper 'Single machine scheduling problems with financial resource constraints: Some complexity results and properties' by E.R. Gafarov et al. 

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#### Abstract

This note emends an incorrectness in the NP-hardness proof of problem $1\left|N R, d_{j}=d, g_{j}=g\right| \sum T_{j}$ given in a paper by Gafarov et al. in Mathematical Social Sciences (see vol. 62, 2011, 7-13).


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In the article E.R. Gafarov, A.A. Lazarev, F. Werner, Single machine scheduling problems with financial resource constraints: Some complexity results and properties, Mathematical Social Sciences, 62 (2011), 7-13, the following mistake is found in Section 3.2 , where the authors consider the problem denoted as $1 \mid N R$, $d_{j}=d, g_{j}=g \mid \sum T_{j}$ and claim that it is NP-hard. In the proof, a reduction from the Partition Problem was used which is not polynomial, since $M$ exponentially depends on $n$.

However, it is not difficult to correct this proof. The main idea of using $M^{n-i+1}$ was that the processing time of a job
belongs to a pair with the smallest number being greater than the total sum of the processing times of all jobs from the pairs with larger numbers, e.g., for the job $V_{2}: p_{2} \gg \sum_{i=2}^{n}\left(p_{2 i-1}\right.$ $\left.+p_{2 i}\right)$.

Instead of using $p_{2 i}=M^{n-i+1}$, where $M=\left(n \sum b_{j}\right)^{n}$ (see the definition of the instance given in (3) on page 11), we can consider, e.g., $p_{2 i}=2 n \cdot 2^{n-i+1} M$, where $M=\left(n \sum b_{j}\right)$. In this case, the reduction will be polynomial in the input length, if we suppose that all digits used are coded in a binary system with approximately $2^{n}$ zero-one symbols per digit.

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