Automation and Remote Control, Vol. 62, No. 4, 2001, pp. 597–606. Translated from Avtomatika i Telemekhanika, No. 4, 2001, pp. 98–108.
Original Russian Text Copyright © 2001 by Kiselev, Polyak.

Minimization of Overshoot in Linear Discrete-Time Systems via Low-Order Controllers

O. N. Kiselev and B. T. Polyak

Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
Received December 4, 2000

Abstract—For the SISO linear discrete-time control systems, a technique of response optimization was proposed. It is based on introducing a performance function which is the upper bound of the maximum of error magnitude. Its minimization leads to a linear programming problem in the controller coefficients. Importantly, a low-order controller can be designed in this manner. Various generalizations of the problem, including the robust variant, were considered.

1. INTRODUCTION

The problem of optimizing the transient response by a performance index such as "overshoot" has simple engineering implications. The interest to it arose as early as in the 1940s, but ponderable results were obtained only recently [1–6]. M.A. Dahleh and J.B. Pearson in their important paper [1] wrote that "the long-standing unsolved problem of *'determine the compensator that minimizes the maximum amplitude of the error to a step input'* has intrigued control engineers for many years." Their solution to this problem, however, even for simpler plants can result in a high-order controller. Therefore, design of low-order controllers that are optimal in the sense of the maximal error magnitude still remains topical. A precise analytical solution seems unlikely because the performance index depends nonconvexly on the controller parameters.

The present paper suggests another optimality criterion which is the upper bound of the original criterion. Optimization by this criterion exploits the linear programming technique. A similar approach based on replacing the criterion was applied to another problem, attenuation of external perturbations [7, 8].

2. FORMULATION OF THE PROBLEM

Consideration is given to the scalar (single-input single-output) linear time-invariant discrete-time system diagramed in Fig. 1 with $G(z)$ for the plant transfer function

$$G(z) = \frac{P(z)}{Q(z)},$$

where $P(z)$ and $Q(z)$ are the coprime polynomials of the delay operator z:

$$P(z) = p_0 + p_1 z + \cdots + p_r z^r,$$
$$Q(z) = q_0 + q_1 z + \cdots + p_s z^s.$$

We lay stress on the fact that z means just a single delay, $zx(k) = x(k-1)$, which is in certain contradiction with the standard domestic notation [9, 10], but agrees well with the international usage [1–8]. The controller $C(z)$ must be chosen so that under the step input

$$v(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$$

0005-1179/01/6204-0597$25.00

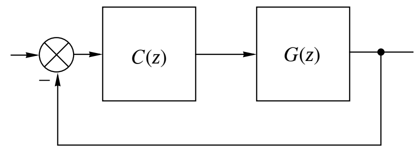

Fig. 1.

and zero initial conditions the magnitude of the system error $e(k)$ be minimal in the criterion

$$J = \|e\|_\infty = \sup_{k\geq 0} |e(k)|. \tag{1}$$

Criterion (1) has a simple physical sense: we need that for all time instants the error be uniformly small. For the most part, $J-1$ coincides with the overshoot—namely, if $J>1$ and the maximum in (1) is attained on $e(k)>0$. It is assumed here that the closed-loop is astatic (has zero static error), that is, $\lim_{k\to\infty} e(k)=0$. This takes place if $z=1$ is a pole of the controller; then, $C(z)$ becomes as follows:

$$C(z) = \frac{A(z)}{(1-z)B(z)},$$

where $A(z)=a_0+a_1z+\dots+a_mz^m$ and $B(z)=b_0+b_1z+\dots+b_nz^n$ are polynomials of z. Additionally, stability of the closed-loop system requires that all roots of the characteristic polynomial

$$D(z) = A(z)P(z) + (1-z)B(z)Q(z)$$

lie outside the unit circle. It follows from

$$E(z) = \frac{1}{1+G(z)C(z)}V(z) = \frac{(1-z)B(z)Q(z)}{D(z)}\frac{1}{(1-z)} = \frac{B(z)Q(z)}{D(z)},$$

where $E(z)$ and $V(z)$ are the z-transforms of the corresponding sequences, that

$$J = \|e\|_\infty = \|E(z)\|_\infty, \tag{2}$$

where the expansion

$$E(z) = e_0 + e_1z + \dots + e_kz^k + \dots$$

holds for the stable rational function $E(z)$ and $\|E(z)\|_\infty$ means that

$$\|E(z)\|_\infty = \sup_k |e_k|.$$

As the result, the problem of error minimization comes to determining the polynomials $A(z)$ and $B(z)$ such that $D(z)$ be stable and criterion (2), minimal. This problem was formulated and solved in [1]. The main disadvantage of this solution is, generally speaking, its infinite dimensionality: the optimum can be attained on the polynomials $A(z)$ and $B(z)$ of unlimited degree. In the same paper, a suboptimal finite-dimensional solution was obtained, but here the degrees of the polynomials can also be prohibitive. Other approaches to this problem were considered in [2–5]. For example, it was proposed in [2–4] to fix the characteristic polynomial $D(z)$, and in [3–4], in particular, $D(z)$ was taken as $D(z)=1$, which gives rise to a FIR (finite impulse response) process, that is, $e(k)=0$ for all sufficiently large k. L.N. Volgin [10] solved the problem of minimizing the duration of the transient process disregarding the error. L. Keel and S.P. Bhattacharyya [5] chose the polynomial $H(z)$ from a family of stable polynomials that were close to the given one. In what follows, we consider another approach to the problem where the characteristic polynomial is not assigned in advance.

3. METHOD OF SOLUTION

We first recall the notion of the superstable polynomial [8, 11]. The polynomial

$$D(z) = d_0 + d_1 z + \cdots + d_\ell z^\ell$$

is regarded as superstable if $|d_0| > \sum_{i=1}^{\ell} |d_i|$. All its roots lie outside the unit circle, and the difference equations of the form

$$D(z)x(k) = v(k)$$

have many convenient features—in particular, one can estimate $\max_k |x(k)|$ under bounded $v(k)$ and initial conditions (see [8] for more detail). Therefore, it is advisable that the characteristic polynomial be superstable, which can be done by an appropriate choice of the controller.

It is difficult to deal directly with criterion (2) because it is not convex like the function of the controller coefficients. We construct its upper bound which is free of this disadvantage. For the rational function

$$a(z) = a_0 + a_1 z + \cdots + a_k z^k + \ldots,$$

we introduce

$$\|a\|_\infty = \sup_{k \geq 0} |a_k|, \quad \|a\|_1 = \sum_{i=0}^{\infty} |a_i|$$

and assume below that all these norms are finite.

Lemma. *The following inequalities for the norms hold:*

(a) $\|ab\|_\infty \leq \|a\|_\infty \|b\|_1$,

(b) $\|ab\|_1 \leq \|a\|_1 \|b\|_1$,

(c) $\left\| \dfrac{1}{1+b} \right\|_1 \leq \dfrac{1}{1 - \|b\|_1}$ *for* $\|b\|_1 < 1$,

(d) $\left\| \dfrac{a}{1+b} \right\|_\infty \leq \dfrac{\|a\|_\infty}{1 - \|b\|_1}$ *for* $\|b\|_1 < 1$.

The lemma is proved in the Appendix. Assertion (d) shows that instead of criterion (2) one can take its upper bound

$$\Psi = \frac{\|BQ\|_\infty}{1 - \|D - 1\|_1} \geq J, \tag{3}$$

which has better properties. It is assumed here that $D(0) = 1$ and $\|D - 1\|_1 < 1$. Therefore, we come to the following approach to the problem.

Algorithm 1.

1. Normalize the plant and controller so that $a_0 p_0 + b_0 q_0 = 1$ and choose the desired orders m and n of the controller's numerator and denominator, respectively.

2. For each $0 \leq \gamma < 1$, determine

$$\begin{gathered} \Phi(\gamma) = \min_{A,B} \|BQ\|_\infty / (1 - \gamma), \\ a_0 p_0 + b_0 q_0 = 1, \\ \|AP + (1 - z)BQ - 1\|_1 \leq \gamma. \end{gathered} \tag{4}$$

For a fixed γ, this problem can be cast as that of linear programming (see the next section). If here the admissible set is empty for all $\gamma < 1$, then one has to return to item 1 and increase m and/or n because it is impossible to attain superstability of the characteristic polynomial for the controllers of the chosen order.

3. If there is no solution to (4) for $0 \le \gamma_0 \le \gamma < 1$, then we determine $\gamma^* = \arg \min_{\gamma_0 \le \gamma < 1} \Psi(\gamma)$; let A^*, B^* be the solution of (4) for this γ.

Theorem. *Let A^*, B^*, and $\Psi^* = \Psi(\gamma^*)$ be obtained by Algorithm 1. Then, for the controller*

$$C^*(z) = \frac{A^*(z)}{(1-z)B^*(z)}$$

the estimate

$$J = \|e\|_\infty \le \Psi^*$$

is guaranteed.

Therefore, we have designed a controller with the given respective orders m and n of the numerator and denominator for which the error magnitude at any time instant does not exceed Ψ^*. We note that among all controllers of the given order generating the superstable characteristic polynomial of the closed-loop system we have determined the minimum of the upper bound (3) of the chosen process performance. Better values of the criterion can be obtained by increasing the controller degree. The designer can find a trade-off between complexity of the controller and its performance values.

4. NUMERICAL ASPECT

We first demonstrate that for a fixed γ problem (4) is indeed reducible to linear programming. We denote by x the vector $x = (a_0, a_1, \ldots, a_n, b_0, b_1, \ldots, b_m)$ of the unknown controller coefficients. Then, the coefficients of the polynomials BQ and $AP + (1-z)BQ - 1$ are linear functions of x, and for some vectors η_k and μ_i and scalars α_k and β_i the problem takes the form

$$\min_x \max_{1\le k\le K} |(\eta_k, x) + \alpha_k|,$$
$$(\mu_0, x) = 1,$$
$$\sum_{1\le i\le I} |(\mu_i, x) + \beta_i| \le \gamma.$$

If additional variables t and s_i are introduced, then it can be rearranged in

$$\min t,$$
$$(\mu_0, x) = 1,$$
$$-t \le (\eta_k, x) + \alpha_k \le t, \quad k = 1, \ldots, K,$$
$$-s_i \le (\mu_i, x) + \beta_i \le s_i, \quad i = 1, \ldots, I,$$
$$\sum_{i=1}^{I} s_i \le \gamma,$$

that is, the standard form of linear programming.

We note that the special iterative methods proposed in [11] for similar problems could have been used here instead of this reduction. In this paper, all numerical results were obtained using the standard program of linear programming from the MATLAB software package. We present some simple examples.

Example 1 ([2]). Consideration is given to a plant of the first order:

$$G(z) = \frac{z - 0.5}{z - 0.2}.$$

By the technique of [1], a suboptimal controller of the ninth order (differing from the optimal one at most by 5%) was designed for which

$$J = \|e\|_\infty = 2.723.$$

Understandably, it is somewhat unnatural to use a controller of the ninth order for a plant of the first order. The results of Algorithm 1 are condensed in Table 1.

Table 1

n	m	Ψ^*	γ^*	$\|e\|_\infty$
1	0	6.67	0	6.67
2	1	3.92	0	3.92
3	2	3.19	0	3.19
4	3	2.91	0	2.91

One can see that the controller of the fourth order

$$C(z) = \frac{-3.45 + 0.36z + 0.73z^2 + 1.45z^3 + 2.91z^4}{(1 - z)(3.63 + 3.61z + 3.49z^2 + 2.91z^3)}$$

provides a performance index which is inferior to that provided by the controller of the ninth order less than by 10%. Interestingly, in this example the optimal γ is zero for all variants of the controller, that is, the characteristic polynomial is unity and the closed-loop system is FIR. Here, the upper bound of the performance index coincides with its true value, that is, $\Psi^* = \|e\|_\infty$. We also note that $J - 1$ defines here the value of overshoot.

Example 2. The object

$$G(z) = \frac{z^2 - 0.9z - 1.6}{z^2 - 0.5z + 1}$$

of the second order is controlled by the following controller of the first order:

$$C(z) = \frac{a_0 + a_1 z}{(1 - z)b_0}.$$

In this case, it is impossible to obtain a characteristic polynomial $D(z)$ equal to unity (for $\gamma = 0$). The criterion $\Psi^* = 0.295$ is minimized for $a_0 = -0.52$, $a_1 = 0.13$, $b_0 = 0.16$ and the optimal value $\gamma^* = 0.45$, and the transient process is of infinite duration.

5. SOME GENERALIZATIONS

It deserves noting that Algorithm 1 can provide a trivial result if $B^* \equiv 0$ and, correspondingly, $\Phi^* \equiv 0$. For example, if the plant is superminimal-phase (if P is a superstable polynomial), then the solution $A(z) = 1$, $B(z) = 0$, that is, the infinite-gain controller, results in complete attenuation of the error: $\min J = 0$. In such situations, performance index (2) loses its sense. Additionally, it loses sense if there is no overshoot or, more precisely, if $J < 1$. It is then recommendable to turn to another performance index:

$$\Theta = \|(1 - \lambda)E(z) + \lambda H(z)\|_\infty, \quad 0 \le \lambda \le 1, \quad \lambda \ne 0.5, \tag{5}$$

where $H(z) = \dfrac{G(z)C(z)}{1 + G(z)C(z)} V(z)$ is the system output. As in the theory of H^∞-design [12], this criterion makes use of the sensitivity function $S(z) = \dfrac{1}{1 + G(z)C(z)}$ and complementary sensitivity function $T(z) = \dfrac{G(z)C(z)}{1 + G(z)C(z)}$, $S(z) + T(z) \equiv 1$.

We will explain the physical implications of criterion (5). Let $H(z) = h_0 + h_1 z + \ldots + h_n z^n + \ldots$; then,

$$\Theta = \|(1 - \lambda)(1 - h_n) + \lambda h_n\|_\infty = \|1 - \lambda + h_n(2\lambda - 1)\|_\infty.$$

For $h = \dfrac{1 - \lambda}{1 - 2\lambda}$, the function $\varphi(h) = |1 - \lambda + h(2\lambda - 1)|$ vanishes. Hence, the functional Θ vanishes for discrete functions h_n with the maximal value $h_{\max} = \dfrac{1 - \lambda}{1 - 2\lambda}$. For small λ, minimization of criterion (5) leads to aperiodic transient processes with small overshoot—for example $h_{\max} = 1.125$ for $\lambda = 0.1$,—which is required in practical use because the monotone processes are too overextended and great values of $h_{\max}$ result in oscillations.

Now, the controller

$$C(z) = \frac{A(z)}{B(z)} \tag{6}$$

must be chosen so as to minimize Θ, that is, to minimize the maximal value of the magnitude of the linear combination of error and system output for the given λ. Since

$$E(z)(1 - \lambda) + \lambda H(z) = \frac{B(z)Q(z)(1 - \lambda) + \lambda A(z)P(z)}{B(z)Q(z) + A(z)P(z)} \frac{1}{1 - z},$$

we required that $z = 1$ be zero of this expression, thus achieving the zero statistical error of the corresponding sequence, that is, satisfaction of the equality

$$B(z)Q(z)(1 - \lambda) + \lambda A(z)P(z) = (1 - z)F(z) \tag{7}$$

for some polynomial $F(z)$. Now, we construct the following algorithm.

Algorithm 2.

1. Determine λ from the admissible overshoot h_{max}: $\lambda = \dfrac{h_{max} - 1}{2h_{max} - 1}$.

2. Normalize the plant and the controller so that

$$a_0 b_0 \lambda + b_0 q_0 (1 - \lambda) = 1$$

and choose the desirable orders m and n of the numerator and denominator of controller (6).

3. From equality (7), determine the polynomial $F(z)$ by comparing the coefficients of the identical degrees z in the left and right sides of the equality, the coefficients of F being dependent here linearly on the coefficients of A and B.

4. For each $0 \le \gamma < 1$, determine

$$\begin{gathered} \Phi(\gamma) = \min_{A,B} \|F\|_\infty / (1 - \gamma), \\ a_0 p_0 \lambda + b_0 q_0 (1 - \lambda) = 1, \\ \|AP\lambda + BQ(1 - \lambda) - 1\|_1 \le \gamma. \end{gathered} \tag{8}$$

For fixed λ and γ, this problem comes to a problem of linear programming. If the admissible set $\Phi(\gamma)$ is empty for all $\gamma < 1$, then one must return to item 2 and increase m and/or n.

5. If a solution of (8) exists for $0 \leq \gamma_0 \leq \gamma < 1$, then we determine $\gamma^* = \arg \min\limits_{\gamma_0 \leq \gamma < 1} \Phi(\gamma)$ and its corresponding controller

$$C^*(z) = \frac{A^*(z)}{B^*(z)}$$

providing the minimal value of the criterion

$$\Phi^* = \Phi(\gamma^*).$$

Theorem 2 where Θ is substituted for criterion J is valid for Algorithm 2.

Example 3. Let us consider the design of the plant

$$G(z) = \frac{z-2}{z-0.5}.$$

We first note that this plant is unstable, but minimal-phase. Attempts to design the controller by criterion (2) using Algorithm 1 lead to $B(z) \equiv 0$. Then, we turn to criterion (5) and by Algorithm 2 find controllers $C(z)$ of the form (6) of different orders for $\lambda = 0.1$.

The results of calculations are condensed in Table 2. Its last column contains the estimates Φ_0 for the FIR processes ($\gamma = 0$). As can be seen from Table 2, optimization in γ provides a better result for all the considered controllers, although the processes here are of infinite duration.

Table 2

n	m	γ^*	Φ^*	Φ_0
1	1	0.36	0.189	0.733
2	2	0.24	0.145	0.283
3	3	0.17	0.128	0.152

Figure 2 depicts the profile of the estimate Φ vs. γ (solid line) and for comparison, the result of direct modeling (dashed line) with a controller of the third order. For $\gamma^* = 0.17$, the estimate Φ^* exceeds the true value of the criterion by 16%. As can be seen from Fig. 2, the estimate Φ and the modeled curve attain their minima virtually for the same value of γ. Stated differently, the designed controller is optimal in criterion (5).

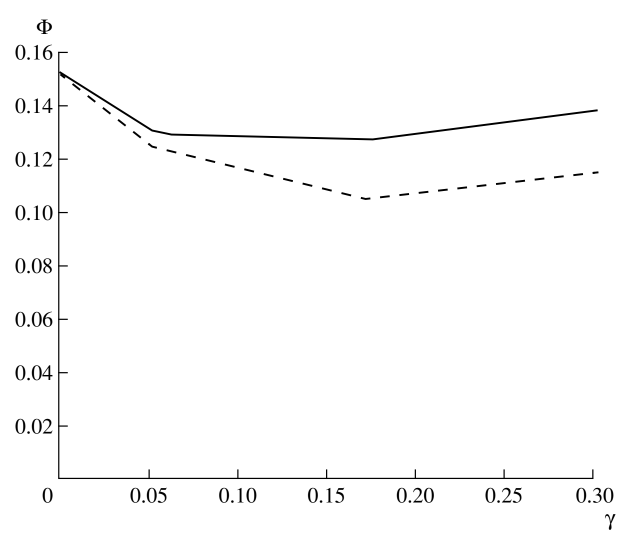

Fig. 2.

6. ROBUST CONTROL

Let us consider the above problem of optimization of criterion (1) for the case where the description of the plant has some uncertainty. Let the transfer function be as follows:

$$G(z) = \frac{P(z) + \delta P(z)}{Q(z) + \delta Q(z)},$$

where $P(z)$ and $Q(z)$ are the given polynomials and $\delta P(z)$ and $\delta Q(z)$, perturbations. Similar to some publications on robust stability and control for discrete-time systems [8, 11, 13–14], we assume that the perturbations are bounded in the ℓ_1–norm:

$$\|\delta P\|_1 \le \varepsilon_1, \quad \|\delta Q\|_1 \le \varepsilon_2 \tag{9}$$

and the degrees of the polynomials δP and δQ can be arbitrary, but need not have constant terms, that is, $\delta P(0) = \delta Q(0) = 0$. By taking into account that now

$$E(z) = \frac{B(z)(Q(z) + \delta Q(z))}{D(z) + \delta D(z)},$$

where $\delta D(z) = A(z)\delta P(z) + (1 - z)B(z)\delta Q(z)$, instead of (3) we obtain on the strength of the Lemma that

$$J \le \Upsilon = \frac{\|BQ\|_\infty + \varepsilon_2\|B\|_\infty}{1 - \|D - 1\|_1 - \varepsilon_1\|A\|_\infty - \varepsilon_2\|(1 - z)B\|_\infty}$$

on the assumption that the denominator is positive and $D(0) = 1$

For the problem of minimizing the maximum of the error magnitude in the presence of ℓ_1-uncertainty (9) in the description of the plant, it is, therefore, possible to minimize the function Υ, the upper bound of the performance index. In this case, Algorithm 1 is somewhat modified, and the problem comes to that of linear programming.

Algorithm 3.

1. Normalize the object and controller so that $a_0b_0 + b_0q_0 = 1$. Choose the desirable orders n and m of the numerator and denominator of the controller.

2. For each $0 \le \gamma < 1$, determine

$$\begin{gathered}\Upsilon(\gamma) = \min_{A,B}(\|BQ\|_\infty + \varepsilon_2\|B\|_\infty)/(1 - \gamma), \\ a_0p_0 + b_0q_0 = 1, \\ \|AP + (1 - z)BQ - 1\|_1 + \varepsilon_1\|A\|_\infty + \varepsilon_2\|(1 - z)B\|_\infty \le \gamma.\end{gathered} \tag{10}$$

For fixed γ, this problem is also reducible to linear programming. If the admissible set is empty for all $\gamma < 1$, then one must return to item 1 and increase n and/or m.

3. If for $0 \le \gamma_0 \le \gamma < 1$ there exists solution of (10), then we determine $\gamma^* = \arg\min_{\gamma_0 \le \gamma < 1} \Upsilon(\gamma)$; then A^*, B^* is the solution of problem (10) for this value of γ.

Algorithm 2 undergoes a similar modification upon using criterion (5).

Example 4. We consider the problem of Example 3 with the controller of the first order of form (7) under uncertainty in the plant: $\|\delta P(z)\| \le 0,1$; $\|\delta Q(z)\| \le 0.05$. Algorithm 3 here also provides the trivial result $B(z) \equiv 0$. Solution is obtained by the modified Algorithm 2 for $\lambda = 0.1$. Uncertainty increased the optimal value of the criterion $\Upsilon^* = 0.211$, $\gamma^* = 0.36$ (see the first row in Table 2) and additionally resulted in the lack of solution for $\gamma = 0$.

The optimal robust controller of the first order is as follows:

$$C(z) = \frac{0.488 + 0.211z}{0.047 + 0.109z}.$$

7. CONCLUSIONS

A technique for response optimization in the linear SISO discrete-time control systems is proposed. It is based on a new optimality criterion, the upper bound of the maximal magnitude of the process error. The resulting problem of optimization turns out to be convex, its solution being reducible to that of linear programming. Consideration also was given to different variants of the criterion and the robust formulation of the problem. Calculations bore out efficiency of the method and the possibility of designing low-order controllers with small overshoot. Generalization of the proposed approach to the MIMO case is still an unsolved important problem.

APPENDIX

Proof of Lemma. For the function $c = ab$ with the coefficients c_k, we have $c_k = a_0b_k + a_1b_{k-1} + \cdots + a_kb_0$. Therefore, $|c_k| \leq \max\limits_{0\leq i\leq k} |a_i| \sum\limits_{i=0}^{k} |b_i| \leq \|a\|_\infty \|b\|_1$, which prove (a). Since $|c_k| \leq |a_0||b_k| + \cdots + |a_k||b_0|$, we obtain by summing these inequalities from 0 to ∞ in k that $\sum\limits_{k=0}^{\infty} |c_k| \leq |a_0| \sum\limits_{i=0}^{\infty} |b_i| + \cdots + |a_k| \sum\limits_{i=0}^{\infty} |b_i| \leq \sum\limits_{k=0}^{\infty} |a_k| \sum\limits_{i=0}^{\infty} |b_i|$, that is, $\|c\|_1 \leq \|a\|_1 \|b\|_1$. By applying this inequality successively for $a = b$, we obtain $\|b^k\|_1 \leq \|b\|_1^k$ and for $\|b\|_1 < 1$ get $\left\|\dfrac{1}{1+b}\right\|_1 = \|1 - b + b^2 - \ldots\|_1 \leq 1 + \|b\|_1 + \|b\|_1^2 + \cdots = \dfrac{1}{1 - \|b\|_1}$. By using successively (a) and (c), we get finally $\left\|\dfrac{a}{1+b}\right\|_\infty \leq \|a\|_\infty \left\|\dfrac{1}{1+b}\right\|_1 \leq \dfrac{\|a\|_\infty}{1 - \|b\|_1}$.

Proof of Theorem. To normalize $a_0p_0 + b_0q_0 = 1$, we have $D(0) = 1$. Therefore, if

$$\|1 - D\|_1 \leq \gamma < 1,$$

then the polynomial $D(z)$ will be superstable. By virtue of the Lemma, we obtain from the formulas

$$\|e\|_\infty = \|E\|_\infty = \left\|\frac{BQ}{D}\right\|_\infty, \quad D = AP + (1 - z)BQ,$$

that

$$\left\|\frac{BQ}{D}\right\|_\infty \leq \frac{\|BQ\|_\infty}{1 - \|1 - D\|_1} \leq \frac{\|BQ\|_\infty}{1 - \gamma}.$$

Therefore, for any $0 \leq \gamma < 1$ and any A and B satisfying

$$\|AP + (1 - z)BQ - 1\|_1 \leq \gamma,$$

the inequality

$$\|e\|_\infty \leq \frac{\|BQ\|_\infty}{1 - \gamma}$$

is valid. By minimizing the right sides first in the admissible A and B and then in all possible γ (as in Algorithm 1), we prove the theorem.

REFERENCES

1. Dahleh, M.A. and Pearson, J.B., Minimization of a Regulated Response to a Fixed Input, *IEEE Trans. Autom. Control*, 1988, vol. 33, no. 10, pp. 924–930.
2. Moore, K.L. and Bhattacharyya, S.P., A Technique for Choosing Zero Locations for Minimal Overshoot, *IEEE Trans. Autom. Control*, 1990, vol. 35, no. 5, pp. 577–580.
3. Casavola, A. and Mosca, E., Minimization of a Closed-Loop Response to a Fixed Input for SISO Systems, *IEEE Trans. Autom. Control*, 1997, vol. 42, no. 11, pp. 1581–1587.
4. Hill, R.D. and Halpern, M.E., Minimum Overshoot Design for SISO Discrete-Time Systems, *IEEE Trans. Autom. Control*, 1992, vol. 38, no. 1, pp. 155–158.
5. Keel, L. and Bhattacharyya, S.P., Robust Stability and Performance with Fixed-Order Controllers, *Automatica*, 1999, vol. 35, no. 10, pp. 1717–1724.
6. Halpern, M.E. and Polyak, B.T., Optimal Tracking with Fixed-Order Controllers, *39th CDC*, Sydney, Australia, 2000, pp. 3908–3913.
7. Blanchini, F. and Sznaier, M., A Convex Optimization Approach to Fixed-Order Controller Design for Disturbance Rejection in SISO Systems, *IEEE Trans. Autom. Control*, 2000, vol. 45, no. 4, pp. 784–789.
8. Polyak, B.T. and Halpern, M.E., The Use of a New Optimization Criterion for Discrete-Time Feedback Controller Design, *38th CDC*, Phoenix, Arizona, 1999, pp. 894–899.
9. Tsypkin, Ya.Z., *Teoriya lineinykh impul'snykh sistem* (Theory of Linear Pulse Systems), Moscow: Fizmatgiz, 1963.
10. Volgin, L.N., *Optimal'noe diskretnoe upravlenie dinamicheskimi sistemami* (Optimal Discrete Control of Dynamic Systems), Moscow: Fizmatgiz, 1986.
11. Vishnyakov, A.N. and Polyak, B.T., Design of Low-Order Controllers for Discrete-Time Control Systems under Nonrandom Perturbations, *Avtom. Telemekh.*, 2000, no. 9, pp. 112–119.
12. Zhou, K., Doyle, J.C., and Glover, K., *Robust and Optimal Control*, Upper Saddle River: Prentice Hall, 1996.
13. Polyak, B.T. and Halpern, M.E., Robust Stability and Design of Linear Discrete-Time SISO Systems under ℓ_1 Uncertainties, *IEEE Trans. Autom. Control*, 1999, vol. 44, no. 11, pp. 2076–2080.
14. Tsypkin, Ya.Z. and Polyak, B.T., Stability of Difference Linear Equations with Unmodelled Higher Order Terms, *J. Differ. Equat. Appl.*, 1998, vol. 3, no. 5/6, pp. 539–546.

This paper was recommended for publication by A.P. Kurdyukov, a member of the Editorial Board